CHAPTER 8 PROJECT

INVESTIGATING NUMBER SEQUENCES

Number theory is sometimes studied simply for the love of math. There is a beauty in finding patterns in sequences of numbers and learning how these sequences overlap with different areas of study. In this project, you will investigate two sequences of numbers and learn how their patterns interact with each other and the golden ratio.

1. The Fibonacci numbers form a sequence of numbers that have a distinct pattern. The first six values of the Fibonacci numbers are as follows.

$$0, 1, 1, 2, 3, 5, \ldots$$

Write a description for the pattern that forms these Fibonacci numbers. (**Hint:** Write sentences to describe how to obtain the next four values in the sequence if the first two values are given.)

2. Using the pattern described in part 1, write the first 15 numbers of the Fibonacci numbers.

3. The notation for each value in the Fibonacci sequence is F_n where n is nth term of the sequence, starting at $n = 0$. This means that $F_0 = 0$ and $F_1 = 1$. One way to think of the Fibonacci sequence is that the pattern described in part 1 is "seeded" with the values $F_0 = 0$ and $F_1 = 1$. Describe the sequence Y_n that would be created with the same pattern if it were seeded with the values $Y_0 = 1$ and $Y_1 = 0$. (**Hint:** List out the first 10 digits of the sequence.)

4. The Lucas numbers L_n form a sequence of numbers that share the same pattern as the Fibonacci numbers, but the sequence is seeded with $L_0 = 2$ and $L_1 = 1$. List out the first 15 numbers of the Lucas numbers. (**Hint:** Use the pattern that was described in part 1.)

5. Compare the Fibonacci numbers with the Lucas numbers. Are any values the same? Do any values match up exactly between the two sequences (that is, does $F_n = L_n$ for any values of n)? Which sequence seems to grow in size the fastest?

6. The *golden ratio* is a mathematical property that has been studied since 300 B.C. Perform an internet search and write a short description of the golden ratio. Include the value of the golden ratio rounded to the nearest millionth. Be sure to describe the use of the golden ratio.

7. Find the ratio of the two largest values of the Fibonacci numbers that were listed in part 2. Find the ratio of the two largest Lucas numbers that were listed in part 4. Round the ratios to the nearest millionth. Compare these two ratios to the value of the golden ratio. Does either the Fibonacci ratio or the Lucas ratio match the golden ratio? How can we improve our investigation into whether each series satisfies the golden ratio?

8. Pick two new seeds for the pattern described in part 1 and list the first 15 values.

9. Compare the values of the new sequence of numbers you created in part 8 to both the Fibonacci numbers and the Lucas numbers. Are any values the same? Do any values match up exactly? Does the new sequence grow faster or slower than the Fibonacci numbers and Lucas numbers?

10. Find the ratio of the two largest values of the new number sequence you created in part 8. Round the ratio to the nearest millionth. Compare this value to the golden ratio. Does it match the golden ratio? Does it match the golden ratio any better or worse than the Fibonacci numbers and Lucas numbers?

11. Do you think that using different seed values for the pattern described in part 1 would result in a ratio that matches closer to the golden ratio? Explain your reasoning.