## 8.1 PROJECT

# PRIME FACTORIZATION AND THE NUMBER OF FACTORS

In Section 8.1, you learned about prime and composite numbers. In this project, you will explore a way to determine the number of factors a number has by using its prime factorization.

1. Find the prime factorization of 18. How many times does each prime factor appear in the prime factorization? Find all possible factors of 18. How many total factors are there?
2. Find the prime factorization of 100, and then find all factors of 100. How many times does each prime factor appear in the prime factorization? How many total factors are there?

It's possible that in parts 1 and 2 you determined the total number of factors by using a brute-force method to list each factor. Let's explore a systematic way to determine the total number of factors.

3. Does any factor in the list of factors of 18 have prime factors that are not listed in the prime factorization of 18? Similarly, does any factor in the list of factors of 100 have prime factors that are not listed in the prime factorization of 100? Explain why it is or isn't possible for the factors to have a prime factor that is not listed in the prime factorization.

Consider the number 72, which has a prime factorization of $2 \cdot 2 \cdot 2 \cdot 3 \cdot 3$. We can create the factors of 72 by choosing how many 2s and 3s we wish to have in the prime factorization of each individual factor.

4. Find the factors of 72 that have no 2s in their prime factorization.
5. Find the factors of 72 that have exactly one 2 in their prime factorization.
6. Find the factors of 72 that have exactly two 2s in their prime factorization.
7. Find the factors of 72 that have exactly three 2s in their prime factorization.

If you combine the factors found in parts 4 through 7 into a list, you will have all of the factors of 72.

8. How many factors appear in each set of factors in parts 4 through 7? How does this number compare to the number of times 3 appears in the prime factorization of 72?
9. How many sets of factors did we find in parts 4 through 7? How does this compare to the number of times 2 appears in the prime factorization of 72?
10. Use the answers to parts 8 and 9 to create an algebraic expression to describe how the total number of factors of 72 compares to the number of 2s and number of 3s in the prime factorization of 72.
11. Can you use the same logic to find an algebraic expression to describe how the total number of factors compares to the prime factorizations of 18 and 100? If so, write an expression for each.
12. Find a general rule that gives the number of factors of a given number based on its prime factorization. (**Hint:** Let $p_1$, $p_2$, and so on represent the prime factors in the prime factorization.)
13. Find the number of factors of 900 using the prime factorization. Compare using the formula to the brute-force method of listing every factor.